

# A Comparative Study between the Pseudo Zernike and Krawtchouk Invariants Moments for Printed Arabic Characters Recognition

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**Abstract**—In this paper, we are focused on characters recognition, for this we present a comparison between the Krawtchouk Invariant Moment (KIM) and the Pseudo Zernike Invariant Moment (PZIM) for the recognition of printed Arabic characters (translated, rotated and contaminated by noise). In the preprocessing phase, we use the thresholding technique, and in the learning-classification phases, we use the supports vectors machines (SVM). The simulation results demonstrates that the KIM method gives more significant results that the PZIM for each Arabic character.

**Index Terms**—Recognition, Printed Arabic characters, Krawtchouk invariant moments, Pseudo Zernike invariant moments, Support vectors machines.

## I. INTRODUCTION

In the pattern recognition many mathematical tools used to extract the primitives from pattern such the moments [1,2], and to train and classify these patterns such a support vector machines (SVM) [3,4].

In fact the moments are efficient methods to extract the primitives from the images that can be learned and classified by the SVM method. In this paper we are focused on the comparison between the performance of the KIM and the PZIM in the recognition of noisy printed Arabic characters. In the preprocessing of the images, representing each character, we use the thresholding technique and in the extraction phase of the primitives, we use the KIM [5] and the PZIM [6] which are used to transform the images to the vectors. The transformed images are used as classes of a SVM in the training phase and then for classify the images at the test phase. We describe our processes of recognitions as follow : In the learning phase we use the SVM method, whose the strategy is one against all for optimally separating each image (which modeled by a class characterized by a label which equal to 1) of the learning base to the rest of the other images that is modeled by another class characterized by a label which equal to 1. This separation (maximizing the margin between two classes) is therefore

creating a decision function separating these 2 classes. We have 28 numerals each of them will be used as a class with a label which equal to 1 and the rest of the other characters will be fully accrued in another class with opposite label which equal to -1. So we built 28 decision functions each of them separating a pair of classes (1 and -1) among the 28 pairs. In the classification phase, we calculate the image of the vector which models the character test preprocessed translated, rotated containing a noise by all 28 decisions functions, the recognition will be given to the character whose decision function separating its classes to another class containing the rest of the other character s which gives the largest value among all the calculated values of the 28 images of the test.

In this paper, we present firstly the technique of the Images preprocessing that we used, then we describe the process of extracting of primitives from images by the KIM and the PZIM then we will deduce the performances of those moments with using the SVM in the learning and classification phases.

## II THE PROPOSED SYSTEM

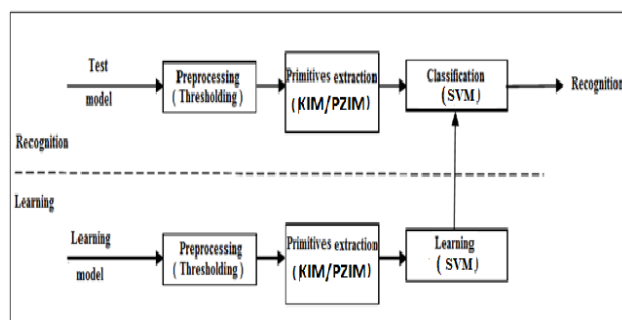


Fig1: The proposed system for Arabic characters recognition.

## III THE PRE-PROCESSING

Before proceeding to the character recognition, it should be to preprocessing the images of these characters

for to eliminate as much as possible the noise.existed in the images. In our approach, we preprocess the images by a thresholding technique which makes the images contain only black and white according a preset threshold.

IV THE PHASE OF PRIMITIVES EXTRACTION

4.1 The Krawtchouk Moments

4.1.1 The Krawtchouk polynomials

The definition of n-th order of classical krawtchouk polynomial is defined as:

$$K_n(x, p, N) = \sum_{k=0}^N a_{k,n,p} x^k = {}_2F_1(-n, -x, -N; \frac{1}{p}) \quad (1)$$

Where :  $x, n = 0, 1, 2 \dots N, N > 0, p \in [0, 1]$ .

${}_2F_1$  is the hyper geometric function[7], defined as :

$${}_2F_1(a, b; c; x) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{x^k}{k!} \quad (2)$$

And  $(a)_k$  is the pochhammer symbol (also rising factorial) defined by:

$$(a)_k = a(a+1)\dots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} \quad (3)$$

The  $\Gamma$  function is defined by:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \quad (4)$$

And :  $\forall n \in N, \Gamma(n+1) = n!$

The set of  $(N+1)$  Krawtchouk polynomials  $\{K_n(x, p, N)\}$  forms a complete set of discrete basis functions with weight function:

$$w(x; p, N) = \binom{N}{x} p^x (1-p)^{N-x} \quad (5)$$

and satisfies the orthogonality condition:

$$\sum_{x=0}^N w(x, p, N) K_n(x, p, N) K_m(x, p, N) = \rho(n, p, N) \delta_{nm} \quad (6)$$

Where :  $m, n = 0, 1, 2 \dots N$ , and  $\rho(n; p, N)$

Is the squared norm, which is given by:

$$\rho(n; p, N) = (-1)^n \left(\frac{1-p}{p}\right)^n \frac{n!}{(-N)_n} \quad (7)$$

And  $\delta_{nm}$  is the Kronecker symbol defined by

$$\delta_{nm} = \begin{cases} 1 & \text{if } n=m \\ 0 & \text{others} \end{cases} \quad (8)$$

4.1.2 The Krawtchouk moments

The Krawtchouk moments have the interesting property of being able to extract local features of an image. The krawtchouk moments of order  $(n+m)$  in terms of weighted krawtchouk polynomials, for an image with intensity function,  $f(x, y)$  is given by:

$$Q_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \bar{K}_n(x; p_1, N-1) \bar{K}_m(y; p_2, M-1) f(x, y) \quad (9)$$

The  $N \times M$  is the number of pixels of an image  $f(x, y)$ . The set

of weighted Krawtchouk polynomials  $\bar{K}_n(x, p, N)$  is defined by :

$$\bar{K}_n(x; p, N) = K_n(x; p, N) \sqrt{\frac{w(x; p, N)}{\rho(x; p, N)}} \quad (10)$$

4.1.3 The Krawtchouk invariants moments

The geometric moments [8,9]of an image  $f(x, y)$  is defined by :

$$M_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} x^n y^m f(x, y) \quad (11)$$

Then the standard set of geometric moment invariants which are independent to rotation, scaling and translation can be written as:

$$V_{nm} = M_{00}^{-\gamma} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} [(x-\bar{x}) \cos \theta + (y-\bar{y}) \sin \theta]^n * [(y-\bar{y}) \cos \theta - (x-\bar{x}) \sin \theta]^m f(x, y) \quad (12)$$

Where :  $\gamma = \frac{n+m}{2} + 1, \bar{x} = \frac{M_{10}}{M_{00}}, \bar{y} = \frac{M_{01}}{M_{00}}$

$$\theta = \frac{1}{2} \arctg \frac{2\mu_{11}}{\mu_{20} - \mu_{02}} \quad (13)$$

And  $\mu_{nm}$  are the central moments defined in as:

$$\mu_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} (x-\bar{x})^n (y-\bar{y})^m f(x, y) \quad (14)$$

The krawtchouk moment invariant is :

$$\tilde{\Omega}_{nm} = \Omega_{nm} \sum_{i=0}^n \sum_{j=0}^m a_{i,n,p_1} a_{j,m,p_2} \tilde{V}_{ij} \quad (15)$$

$$\Omega_{nm} = [\rho(n; p_1, N-1) \cdot \rho(m; p_2, M-1)]^{-1/2} \quad (16)$$

With :  $\tilde{V}_{ij} = \sum_{p=0}^i \sum_{q=0}^j \binom{i}{p} \binom{j}{q} \left(\frac{N^2}{2}\right)^{\frac{p+q}{2}+1} \left(\frac{N}{2}\right)^{i+j-p-q} V_{pq}$  (17)

And :  $\binom{x}{y} = \frac{x!}{y!(x-y)!}$  (18)

For each character, the values calculated by the krawtchouk invariant moments are used as a vectors which will be considered as a class of a SVM which's making in the characters recognition.

4.2 The Pseudo Zernike Moments

For a image  $f(x, y)$ , the pseudo-Zernike moment of order n and repetition m is given by:

$$A_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x, y) V^*(x, y) \quad (19)$$

$$V^*(x, y) = R_{nm}(x, y) e^{jm \arctan(y/x)} \quad (20)$$

$$\text{And: } R_{nm}(x, y) = \sum_{s=0}^{n-|m|} \frac{(-1)^s (x^2 + y^2)^{\frac{n-s}{2}} (2n+1-s)!}{s!(n-|m|-s)!(n+|m|+1+s)!} \quad (21)$$

where  $x^2 + y^2 \leq 1$  and the symbol \* denotes the complex conjugate operator.

4.2.1 The pseudo Zernike invariants moments

The pseudo-Zernike moment is invariant under rotation but sensitive to translation and scale. Therefore a normalization must be done of this moments.

$$f(x, y) = f\left(\bar{x} + \frac{x}{a}, \bar{y} + \frac{y}{a}\right) \quad (22)$$

where  $(\bar{x}, \bar{y})$  being centered of pattern function  $f(x, y)$  and

$a = (\beta/M_{00})^{1/2}$ ,  $\beta$  is a predetermined value for the number of object points in the pattern.

We note that The KIM formula is more complex than that of the PZIM.

V THE LEARNING PHASE

5.1 Principle of Functioning between Two Classes of the SVM

5.1.1 Linear case

Given a set of vectors  $x_i \in \mathcal{R}^n$   $n$  is the dimension of the vector space and two classes. The first class containing a party of these vectors and bears the label 1, the second class contains the other party of vectors and bears the label -1. the goal of SVM [10] is to find a classifier that will separate these classes and maximize the distance between them. This classifier is called hyperplane (see figure 2)

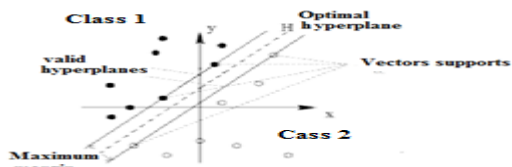


Fig 2: determination of optimal hyperplane, vectors supports, maximum marge and valid hyperplane.

The nearest points which alone are used for determination of hyperplane are called support vectors. The property of SVM is that this hyperplane must be optimal that is to say it must maximize the distance between the supports vectors of a class and those of the other class. The classifier is represented by :

$$f(x, w, b) : x \rightarrow y \quad (23)$$

Where  $w$  and  $b$  are the parameters of the classifier  $y$  is the label.

5.1.2 The primal/dual problems

5.1.3 The primal problem

For to maxim the distance between the supports vectors of a class and those of the other class, we must to solve a problem of minimization under the constraints called the primal problem:

To minimize  $P(w, b) = \frac{1}{2} \|w\|^2$  (24)

Such that  $y_i (wx_i + b) \geq 1$

5.1.4 The dual problem

To simplify the calculations, it is necessary to introduce a formulation called dual of the problem by using the Lagrangian operator  $L$

$$L(w, b, \alpha) = \frac{1}{2} \|w\|^2 - \sum_{i=1}^n \alpha_i [y_i (wx_i + b) - 1] \quad (25)$$

The dual variables  $\alpha_i$  intervening in the Lagrangian are called Lagrange multipliers. The dual problem is:

To maximize  $D(\alpha) = \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j x_i x_j$

Such that  $\sum_{i=1}^n \alpha_i y_i = 0$   $\alpha_i \geq 0, \forall i=1,2,\dots,n$  (26)

Only the  $\alpha_i^*$  corresponding to points nearest to hyperplane is nonzero, we speak of support vectors. The decision function associated is:

$$f(x) = \sum_{i=1}^n \alpha_i^* y_i x_i \cdot x + b \quad (27)$$

5.1.5 Non linear case

In the linear case (see figure 3), the classification of data is easy, but in the nonlinear case the representation space  $\mathcal{R}^n$  must be transformed to a other space of higher dimension is called space of re-description  $\mathcal{R}^p$  ( $p > n$ ), this transformation is carried by virtue a special functions called the kernel functions:

$$K : \mathcal{R}^n \times \mathcal{R}^n \rightarrow \mathcal{R}^p \quad p > n \quad (28)$$

$$(x_i, x_j) \rightarrow K(x_i, x_j)$$



Fig 3: nonlinear separation between class 1 and class 2.

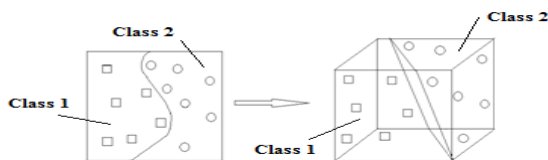


Fig 4: transformation of original data space to a re-description space.

We must solve therefore:

To maximize  $D(\alpha) = \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j K(x_i, x_j)$

Such that  $\sum_{i=1}^n \alpha_i y_i = 0$  (29)

$0 \leq \alpha_i \leq C, \forall i = 1, 2, \dots, n$

The parameter  $C$  which appears here is a positive constant fixed in advance, it's called the penalty constant. The decision function has the form:

$$f(x) = \sum_{i=1}^n \alpha_i^* y_i K(x, x_i) + b \tag{30}$$

Some example of the kernel functions:

Kernel linear	$xy$
Kernel polynomial of degree n	$(axy + b)^n$
Gaussian Radial Basis Function(GRBF):	$e^{-\frac{\ x - y\ ^2}{2\sigma^2}}$

5.2 Principle of Functioning between a Several Classes of the SVMs

The method described above is designed for two classes problem many studies treat a generalization of this method to multi-classification classes [11] among these studies we cite the two most frequently used strategies: The first approach is to use N decision functions (one against t all) allowing to make the discrimination of each class against all others. The decision rule used in this case is usually the maximum such that we assign an unknown vector x into the class associated with the output of SVM is the largest:

$$i = \arg \max_{i=1, 2, \dots, N} f_i(x) \tag{31}$$

The second method called one against one Instead of learning N decision functions, each class is discriminated against another So  $\frac{N(N-1)}{2}$  decision functions are learned

and each of them performs a voting for the assignment of a new unknown vector x. its class then becomes the majority class after the vote.

In our work, We use the kernel function GRBF with the standard deviation  $\sigma = 0.2$  and the penalty constant:  $C = 10^{30}$ .

VI THE CLASSIFICATION PHASE

After having built the 28 decision functions between the 28 pairs of classes in the learning phase by the strategy of(one against all) we calculate the values of the images vector modeling the character test by all the 28 decision functions the recognition will be given to the character whose decision function separating its class to another class containing the rest of the other characters which gives the largest value among all calculated values of the 28 images of the test.

VII EXPEREMENTS AND RESULTS

We choose the size image 50x50. Each character was converted to vector of 7 components which are the KIM/PZIM that used as: a class having a label that is equal to 1, and the rest of all the other vectors modeling the other characters can be used. The separation of the 28 pairs of two classes by the construction of the 28 decision functions is fact by the strategy of one against all. First we present the test character translated, rotated...and not containing a noise, then we add increasingly an quantity

of noise of type ‘salt & pepper‘ for to know the effect of noise added on the rate recognition of each character, for each moment. We choose the parameters of the KIM equals to: p=q=0.96. For each test character noisy, translated, rotated...The noise used is of type ‘salt & pepper’, its values are: [0,0.01,0.02, 0.03, 0.04 ,0.05,0.06, 0.07,0.08, 0.09,0.1,0.11,0.12,0.13,0.14,0.15,0.16, 0.17,0.18, 0.19, 0.2]. We group the results obtained in the following tables:

TABLE 1: THE RECOGNITION RATE OF EACH ARABIC CHARACTER.

Character	$\tau_c$ (PZIM)	$\tau_c$ (KIM)
أ	39%	100%
ب	48%	100%
ت	58%	58%
ث	81%	96%
ج	29%	34%
ح	34%	72%
خ	29%	72%
د	29%	100%
ذ	48%	100%
ر	100%	100%
ز	62%	100%
س	29%	34%
ش	100%	68%
ص	20%	39%
ض	34%	48%
ط	100%	29%
ظ	43%	39%
ع	39%	58%
غ	68%	34%
ف	53%	81%
ق	100%	100%
ك	100%	100%
ل	81%	100%
م	20%	100%
ن	100%	100%
ه	34%	39%
و	48%	100%
ي	29%	100%

TABLE 2 :  
THE VARIATION OF THE GLOBAL RECOGNITION RATE IN FUNCTION OF NOISE ADDED. AND THE ASSOCIATED GRAPH

Noise	$\tau_g$ (PZIM)	$\tau_g$ (KIM)
0	100%	100 %
0.01	100%	100%
0.02	100%	100%
0.03	100%	100%
0.04	93%	100%
0.05	93%	97%
0.06	75%	86%
0.07	65%	75%
0.08	58%	75%
0.09	54%	72%
0.10	43%	72%
0.11	40%	65%
0.12	36%	65%
0.13	36%	61%
0.14	33%	54%
0.15	33%	54%
0.16	33%	54%
0.17	25%	50%
0.18	25%	50%
0.19	25%	50%
0.20	25%	47%

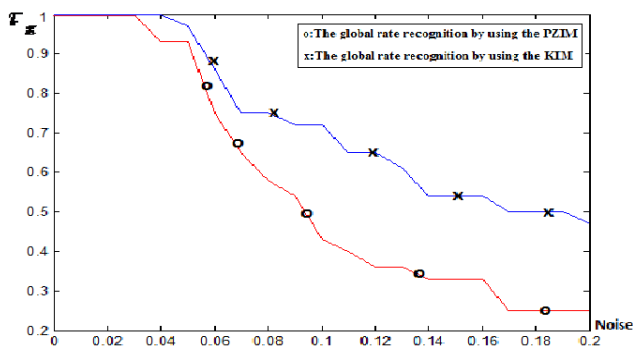


Fig 5: Evolution of global rate recognition  $\tau_g$  of the PZIM and the KIM in function of noise added.

For two moments, the global rate recognition is a decreasing function according to noise added, but the important remark is that the falling of this rate of the PZIM is greater than the rate of the KIM, this that we check that the KIM is more robust than the PZIM against noise.

VIII CONCLUSION

In this paper we have concentrated on character recognition of noisy Arabic characters. The results, shows that reliable recognition is possible using a thresholding technique in the preprocessing phase and the invariants moments of Krawtchouk and those of pseudo Zernike in the primitives' extraction phase and the SVMs in the learning and the classification phase. The simulation result demonstrates that the KIM is more robust and more performing than the PZIM, despite the fact that the recognition time of the KIM is greater than that of the PZIM.

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