3D Normalization Based on the Barycentric Coordinates

El Mostafa Rajaallah
Laboratory LAVETE,
Dept. Mathematics and Computer Science
Faculty of Science and Technology,
University Hassan 1st
Settat, Morocco
m.rajaallah@uh1.ac.ma

Lahcen Moumoun
Laboratory LAVETE,
Dept. Mathematics and Computer Science
Faculty of Science and Technology,
University Hassan 1st
Settat, Morocco
lahcenm@gmail.com

Taoufiq Gadi
Laboratory LAVETE,
Dept. Mathematics and Computer Science
Faculty of Science and Technology,
University Hassan 1st
Settat, Morocco
gtaoufiq@yahoo.fr

***Abstract*—The most common technique in the normalization of 3D objects is the Principal Component Analysis (PCA). However, it is well known that the principal axes generated by the PCA may be different for similar shapes. To overcome the limitations of the PCA we propose in this paper a normalization method to give robustness to remeshing, rotations and reflections of meshed 3D objects. This method is based on the integration of the mesh volume, the use of barycentric coordinates and the CPCA (Continuous Principal Component Analysis).**

***General Terms*—Image Processing.**

***Index Terms*—Normalization, PCA, barycentric coordinates.**

I. INTRODUCTION

Duplicating real-world object in a digital environment was an interesting task for many applications. The quality of the obtained models was often limited by the capacity of the existing hardware and software. However, recent advances in scanning technology and three-dimensional modeling helped to visualize and manipulate complex models with ease. 3D objects are usually given in an arbitrary orientation and scale. To improve the accuracy of the use of these objects, and more particularly the search by content systems, a near-normalization treatment is often necessary. Thus, the normalization of a 3D object is to orient it correctly in a canonical marker.

The normalization method proposed in this paper aims at addressing problems related to meshing resolution and the orientation of the principal axes of normalized objects. Our method is based on an integration of the mesh volume [15], using the barycentric coordinates of the vertex [14] to generate the matrix of moments (order 2). After generating the matrix of two-order moments, we apply the PCA [1] of the matrix to define a canonical marker for the object. At the end, we will consider a reflection coefficient inspired from the CPCA (Continuous Principal Component Analysis) [2].

We present in the beginning of this article an overview on 3D normalization. Next, we describe our proposal to standardize a 3D object. At the end of this article we present our experimental results with a qualitative and quantitative evaluation of the proposed method.

II A OVERVIEW ON 3D NORMALIZATION

The most commonly used technique for normalization of a 3D object is based on Principal Component Analysis (PCA), in which the center of gravity is chosen as the origin, the size of the bounding box as a scale factor of the form and the determining canonical axes based on the calculation of the eigenvalues and eigenvectors of the covariance matrix of the set of points representing the object in question. The eigenvalues are sorted in a descending order, the eigenvectors are chosen on the basis of this ranking, the first vector is aligned with the first axis (x), the second with the second axis (y) and the third vector with the third axis (z) (Paquet et al). [1].

The PCA has been extended by Vranic et al. [2], leading to the PCA continues (Continuous Principal Component Analysis), the proposed approach is more accurate than traditional ACP (discrete), but it is a little more expensive in execution time.

Several Normalization approaches have been developed recently. Ricard et al. [12] proposed a method to integrate a 3D object without using a discrete representation but directly from its bounding. Their method is based on the contour's integration of the object for generating the moments' matrix; it is robust against remeshing of 3D objects. The proposed method uses the discrete PCA, on moments's matrix, with its limitations.

Tedjoknsumo et al. [3] proposed a normalization method based on bilateral symmetry planes (PSB). In their method, they calculate the three axes of the PCA and the three planes normal to its axis. Subsequently, they consider that the plan ACP generates the smallest error of symmetry (a feature introduced by the authors). Then, they pivot this plane around the three axes of rotation with predefined increments to generate the plane which minimizes the error of symmetry. After projecting all the points of the 3D object on this plan, and applying

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PCA 2D on the projected points for generate the first and second main axes, it turned out that their method is time consuming due to the procedure of research the planes of symmetry.

H. Fu et al. [4] presented a solution for detecting the vertical orientation of the object. Their method is based on the assumption that most objects in real life are symmetrical with respect to a plane, but this method is not appropriate when dealing with deformable shapes.

Chaouch et al. [5] proposed a method based on the symmetry properties of Minovic et al. [6] by considering the interesting properties of reflection symmetries of the PCA. The axes of the PCA are considered and the axes for initial's shape are studied. They introduced a measure for assessing local symmetry of translational invariance (CILT) whose main objective is to provide optimal directions (principal axes) for characterization compact and relevant of the 3D shape. The limitation of this method it is based on assumptions derived from human perception.

Yu-Shen et al. [7] proposed a method based on the LMS (Least Median of Squares) by considering the work of Fleishman et al. [8] to guide the calculation of the principal axes of the PCA. The proposed method gives good results for deformed objects. The main limitation of this method is that it is sensitive to the density of the samples.

Recently, as part of a project funded by the National Science Foundation of China, Chao Wang et al. [9] proposed a normalization method articulated volumetric 3D shapes. The main contribution of their work can be summarized in a proposed normalization algorithm to estimate the location and orientation of articulated 3D shapes, based on solving a problem using weighted least squares IRLS (Iteratively Reweighted Least Squares) and the implicit shape representation (IS-Rep: value introduced by the authors). A function of articulation insensitive natural weight is proposed to reduce the influence of the deformation articulated during the standardization process. The limitation of this method is that the orientation of the shape is not clear and is subjected to a large extent shape deformation.

It appears from this study, that there is currently no satisfactory method to both the constraints of normalization's good quality and low complexity. However, the PCA (and CPCA) remains the most adopted approach.

III THE PROPOSED METHOD

In this paper we suggest a normalization method, based on barycentric coordinates, inspired from [2, 12, 14, 15, 16]. Using these coordinates to achieve robustness to rotations and reflections to remeshing of 3D objects.

3.1 Barycentric Coordinates

Definition: we consider a triangle $T = \langle V_1; V_2; V_3\rangle$ non degenerate representing one face of a 3D object, V a point of the tetrahedron D, composed from T and the center of gravity of the object. Consider a bi = bi(V) such as:

$$V = b_1V_1 + b_2V_2 + b_3V_3 \qquad (1)$$

With b_1, b_2 and b_3 the barycentric coordinates of V relative to T.

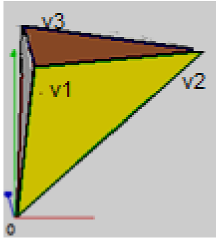

Fig. 1 Tetrahedron built from a surface of a 3D object

Note: The barycentric coordinates for a point V belonging to D with respect to T are unique. Consider a $V=(x, y, z)^T \in R^3$ and $V_i=(x_i, y_i, z_i)^T \in R^3$, with i=1,2,3. Then the system:

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix}\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad (2)$$

admits a unique solution (since the triangle T is non-degenerate). Using Cramer's method we obtain:

$$b1 = \frac{\begin{vmatrix} x & x_2 & x_3 \\ y & y_2 & y_3 \\ z & z_2 & z_3 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}}, b2 = \frac{\begin{vmatrix} x_1 & x & x_3 \\ y_1 & y & y_3 \\ z_1 & z & z_3 \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}}$$

$$et\ b3 = \frac{\begin{vmatrix} x_1 & x_2 & x \\ y_1 & y_2 & y \\ z_1 & z_2 & z \end{vmatrix}}{\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}} \qquad (3)$$

Theorem 1. Let $T = \langle V_1; V_2; V_3\rangle$ and $TR = \langle RV_1;RV_2;RV_3\rangle$ two non-degenerate triangles, with a diagonal matrix R. is a point of the tetrahedron formed by the center of gravity of the 3D object and the triangle T. Let b_i, b_{iR}, with i=1, 2, 3 ; the barycentric coordinates of V on T and TR; satisfy the following equation: $b_{iR}(RV)=b_i(V)$. (4)

The previous theorem shows that the barycentric coordinates are invariant under rotation [14].

3.2 Principal of Our Approach

Normalization parameters of the 3D object are calculated by the integration of the tetrahedra containing the three points of the faces in addition to the center of gravity of the object. The geometrical moments of order 1 are coordinates of the center of gravity g of the object. The alignment is done by calculating the eigenvectors of the moment's matrix M (order 2), respecting the logic of the PCA.

$$g = (M_{100}\ M_{010}\ M_{001}) \qquad (5)$$

$$M=\begin{bmatrix} M_{200} & M_{110} & M_{101} \\ M_{110} & M_{020} & M_{011} \\ M_{101} & M_{011} & M_{002} \end{bmatrix} \qquad (6)$$

To ensure the invariance considerations, we compute the signed distances from the surface of the object for the three planes (YOZ) (ZOX) and (XOY) defined as follows:

$f_x = \iint_S \operatorname{sign}(p_x)|p_x|ds$, , (7)

p =(p_x, p_y, p_z) point of meshed 3D objects.

Ditto for f_y, f_z

Where, p_x, p_y and p_z are respectively the projections of p on the plans (YOZ) (ZOX) and (XOY)

A diagonal matrix defines the reflection matrix:

$$F = \operatorname{diag}(\operatorname{sign}(f_x), \operatorname{sign}(f_y), \operatorname{sign}(f_z)) \quad (8)$$

.

3.3 The Calculation of the Geometrical Moments

D_i is a tetrahedron compound the points g(0, 0, 0), $p_1(x_{i1},y_{i1},z_{i1})$, $p_2(x_{i2},y_{i2},z_{i2})$, $p_3(x_{i3},y_{i3},z_{i3})$, the geometrical moments of order (p+q+r) can be calculated by integrating:

$$m_{pqr} = \int_{Di} x^p y^q z^r dxdydz = \int_{Di} f(x,y,z)dxdydz \quad (9)$$

With $\int_{Di} f(x,y,z)dxdydz = \int_{di} |Ti| f(X,Y,Z)dXdYdZ$

And di =<(0,0,0), (1,0,0), (0,1,0), (0,0,1)> is the orthogonal tetrahedron unit.

Ti is the triangle formed by the points p_1, p_2 et p_3 and X, Y et Z are the barycentric coordinates of V(x, y, z) with respect to Ti=<p_1 ; p_2 ; p_3>.

$$f(X,Y,Z) = (x_{i1}X + y_{i1}Y + z_{i1}Z)^p (y_{i2}X + y_{i2}Y + z_{i2}Z)^q (x_{i3}X + y_{i3}Y + z_{i3}Z)^r \quad (10)$$

The moment of a 3D object can be seen as the sum of the moments tetrahedra compounds and center of gravity of the object and the faces of the meshing of the object [15] [16].

$$m^i_{pqr} = |Ti| \int_{di} f(X,Y,Z)dXdYdZ \quad (11)$$

$$M_{pqr} = \sum_{i=1}^{N_T} m^i_{pqr} \quad (12)$$

With N_T is the number of faces of the mesh and :

$$|Ti| = \begin{vmatrix} x_{i1} & x_{i2} & x_{i3} \\ y_{i1} & y_{i2} & y_{i3} \\ z_{i1} & z_{i2} & z_{i3} \end{vmatrix} \quad (13)$$

therefore

$$\begin{bmatrix} M_{200} & M_{110} & M_{101} \\ M_{110} & M_{020} & M_{011} \\ M_{101} & M_{011} & M_{002} \end{bmatrix} = \sum_{i=1}^{N_T} \begin{bmatrix} m^i_{200} & m^i_{110} & m^i_{101} \\ m^i_{110} & m^i_{020} & m^i_{011} \\ m^i_{101} & m^i_{011} & m^i_{002} \end{bmatrix} \quad (14)$$

.

3.4 Evaluation Measure: Rectilinearity Normalized Objects

Definition 1: A 3D mesh is rectilinear if the angles between each two faces belong to the set {0, π / 2, π, 3π / 2} [19].

Definition: For a 3D object mesh (B) consisting of N triangle {T1, T2, …, TN}, its measurement of rectilinearity:

$$\operatorname{Rect}(B) = \frac{S(B)}{P(B)} \quad (15)$$

and :

- $S(B) = \sum_{i=1}^{N} S(T_i)$ is the sum of surfaces of component triangles of the object..
- $P(B) = P_x + P_y + P_z$
- $P_x = \sum_{i=1}^{N} S(T_{ix})$ is the sum of the surfaces of the projections of the triangles Ti on the plane (ZOY).
- $P_y = \sum_{i=1}^{N} S(T_{iy})$ is the sum of the surfaces of the projections of the triangles Ti on the plane (XOZ).
- $P_z = \sum_{i=1}^{N} S(T_{iz})$ is the sum of the surfaces of the projections of the triangles Ti on the plane (XOY).

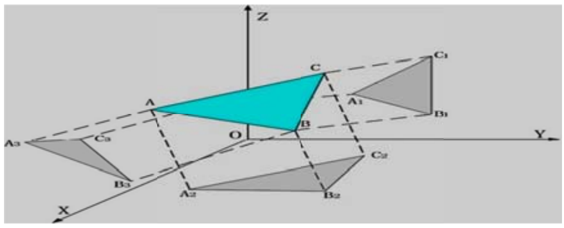

Fig. 2 Projections of a triangle with respect to the three plans [19]

Theorem 2: A 3D mesh (B) is rectilinear if and only if:

$$\operatorname{Rect}(B,\alpha,\beta,\gamma) = \frac{S(B)}{P(B,\alpha,\beta,\gamma)} = 1 \quad , \quad \text{for} \quad \alpha,\beta,\gamma \in [0, 2\pi]$$

angles of rotation relative to the planes (ZOY), (XOZ), (XOY) respectively.

Proof. See [19] page 135.

Theorem 3: For a 3D object meshed (B):

$$\frac{1}{\sqrt{3}} \leq \operatorname{Rect}(B,\alpha,\beta,\gamma) \leq 1$$

Proof. See [19] page 136.

IV Experimental Results

The databases we have used for our tests are based on SHREC'07 [10] and 3D Segmentation Benchmark [17].

SHREC'07 which was created as part of the contest "3D Shape Retrieval Contests", was used to evaluate research methods 3D. The database contains 400 models SHREC'07 triangular mesh in format "OFF" divided into 20 categories (male, glasses, plane ...).

Benchmark "3D Segmentation Benchmark" was created within the framework of the project "3D Models And Dynamic Representation And segmentation models" [18]. The purpose of this benchmark is to provide an automated tool to evaluate, analyze and compare different algorithms for automatic segmentation of 3D meshes.

4.1 Qualitative Evaluation

The main advantages of the PCA are its simplicity and speed. It can be applied to most of the 3D models.

A first limitation of the PCA is that it is not robust to the deformation of objects. Principal axes generated by the PCA may be different for similar shapes. This limitation is illustrated in Figure 3. This figure shows the results of applying PCA to the object "12.off" [10] before and after deformation.

To highlight the invariance of the proposed method compared to deformations, we present in Figure 4, the results obtained by our method for the same object.

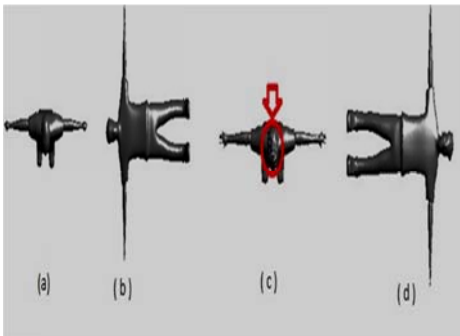

Fig. 3 PCA Normalization (b) for object (a), applying a deformation (c) and normalization after deformation (d)

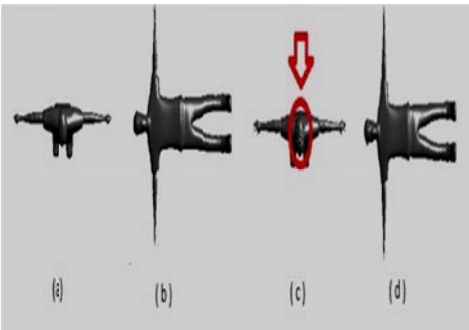

Fig. 4 Barycentric Normalization (b) for object (a), applying a deformation (c) and normalization after deformation (d)

The application of PCA on two clouds of points where the only difference between them is a rotation and / or translation can lead to the same axis directions PCA but not necessarily the same direction [13, 11] (Fig. 5).

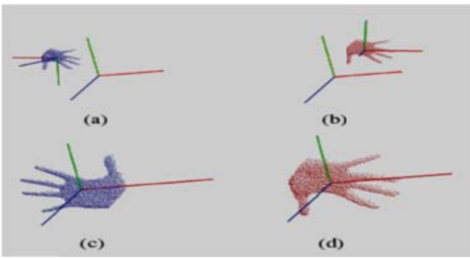

Fig. 5 Problems related to the axis direction of the ACP for the same object that has undergone to rotations [12]

Figures 6 and 7 show a comparison between the results obtained by our method with PCA standard. The object used is "octopus" [17], with and without the application of a 120 ° rotation.

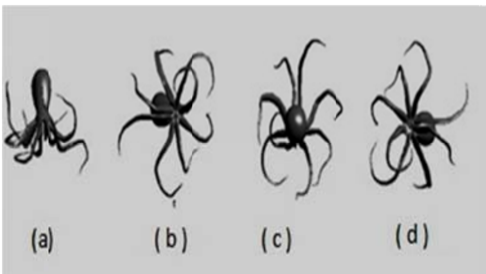

Fig. 6 PCA Normalization (b) for objet (a), applying a 120° rotation (c) and normalization after rotation (d)

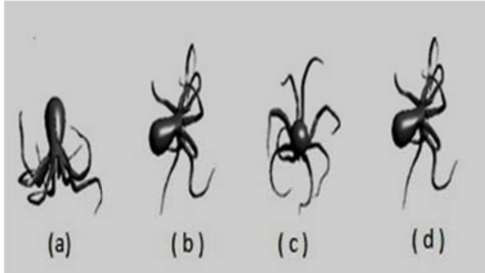

Fig. 7 Barycentric Normalization (b) for objet (a), applying a 120° rotation (c) and normalization after rotation (d)

The obtained results thus show the invariance of the proposed method to rotations. Indeed, whatever the angle of rotation of the initial object is, we obtain, unlike the PCA, the same direction and ditto for axes direction.

4.2 Evaluation of the Rectilinearity for Normalized Objects

To quantitatively evaluate our method, we are based on the criterion of rectilinearity "Rect" presented above. We used for the parameters, α, β and γ, the following values: $\alpha = \pi/3$, $\beta = \pi/3$ et $\gamma = \pi/3$.

Table 1 presents measures of rectilinearity obtained respectively for an initial object, the object after the implementation of the PCA and the object after applying our method. The objects used in this comparison are extracted from the database [17].

TABLE 1
MEASURING RECTILINEARITY RECT

Object	Initial RECT	After ACP Normalization	After Barycentric Normalization
alie	0,674	0,7047	0,7063
armadillo	0,6701	0,6687	0,6705
boy	0,6873	0,7094	0,7094
bunny	0,6698	0,6631	0,6802
homer	0,6678	0,6952	0,6954
robot	0,6818	0,7294	0,7333
vaselion	0,6655	0,6913	0,6925

The results obtained by our method, for objects: alien, armadillo, boy, homer, and robot vaselion are almost similar with respect to PCA. the rectilinearity measurements obtained for the object "bunny" are relatively distinct. For this purpose, our method guarantees more rectilinearity.

V CONCLUSION

Our method can be seen as a hybrid method that is based on the study made by the the integration of the mesh volume [15] and adopted by [16, 12], barycentric coordinates [14] and the CPCA [2].

Extractiing (Extracting) the normaliization's parameters on the surface of the object is independent of the choice of the resolution discretization and provides a normalization independent of the meshing and small deformations. Applying the factor of reflection provides normalized objects independent of any parameter. The limitation of this method is the computational cost, for this reason we plan to improve it in order to overcome this limitation.

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